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## Lutetium(III) cyclotetraphosphate

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Key indicators: single-crystal X-ray study; $T=296 \mathrm{~K}$; mean $\sigma(\mathrm{Lu}-\mathrm{O})=0.003 \AA$; $R$ factor $=0.020 ; w R$ factor $=0.038 ;$ data-to-parameter ratio $=17.5$.

Single crystals of the title compound, tetralutetium(III) tris(cyclotetraphosphate), $\mathrm{Lu}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$, were obtained by solid-state reaction. The cubic structure is isotypic with its $\mathrm{Al}^{\mathrm{III}}$ and $\mathrm{Sc}^{\mathrm{III}}$ analogues and is built up from four-membered $\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)^{4-}$ phosphate ring anions ( $\overline{4}$ symmetry), isolated from each other and further linked through isolated $\mathrm{LuO}_{6}$ octahedra (.3. symmetry) via corner sharing. Each $\mathrm{LuO}_{6}$ octahedron is linked to six $\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)^{4-}$ rings, while each $\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)^{4-}$ ring is linked to eight $\mathrm{LuO}_{6}$ octahedra.

## Related literature

The title compound belongs to a structural type discovered a long time ago through the $\mathrm{Al}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ member, the structure of which was first investigated by Hendricks \& Wyckoff (1927) and then described by Pauling \& Sherman (1937). Since then, five isotypic compounds have been characterized: $\mathrm{Cr}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ (Rémy \& Boullé, 1964); $\mathrm{Ti}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ (Liebau \& Williams, 1964); $\mathrm{Fe}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ (d'Yvoire et al., 1962); $\mathrm{Sc}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ (BagieuBeucher, 1976; Mezentseva et al., 1977; Bagieu-Beucher \& Guitel, 1978; Smolin et al. 1978) and $\mathrm{Yb}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ (Chudinova, 1979). For a review of the crystal chemistry of cyclotetraphosphates, see: Durif (1995). For other polymorphs of composition $\mathrm{Lu}\left(\mathrm{PO}_{3}\right)_{3}$, see: Höppe \& Sedlmaier (2007); Yuan et al. (2008); Bejaoui et al. (2008).

## Experimental

## Crystal data

| $\mathrm{Lu}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ | $Z=4$ |
| :--- | :--- |
| $M_{r}=1647.52$ | Mo $K \alpha$ radiation |
| Cubic, $I \overline{4} 3 d$ | $\mu=13.08 \mathrm{~mm}^{-1}$ |
| $a=14.6920(6) \AA$ | $T=296 \mathrm{~K}$ |
| $V=3171.3(2) \AA^{3}$ | $0.18 \times 0.10 \times 0.08 \mathrm{~mm}$ |

## Data collection

Bruker APEXII CCD
diffractometer
Absorption correction: multi-scan
(SADABS; Bruker, 2008)
$T_{\text {min }}=0.534, T_{\text {max }}=0.746$
3088 measured reflections 717 independent reflections 659 reflections with $I>2 \sigma(I)$ $R_{\text {int }}=0.034$

## Refinement

$R\left[F^{2}>2 \sigma\left(F^{2}\right)\right]=0.020$
$w R\left(F^{2}\right)=0.038$
$\Delta \rho_{\text {max }}=0.90 \mathrm{e}^{\AA^{-3}}$
$\Delta \rho_{\min }=-0.67 \mathrm{e} \AA^{-3}$
717 reflections
41 parameters
Absolute structure: Flack (1983),
272 Friedel pairs
Flack parameter: 0.000 (15)

Table 1
Selected bond lengths ( $\AA$ ).

| $\mathrm{Lu}-\mathrm{O}^{\text {i }}$ | 2.182 (3) | $\mathrm{Lu}-\mathrm{O} 2$ | 2.185 (4) |
| :---: | :---: | :---: | :---: |
| $\mathrm{Lu}-\mathrm{O}^{\text {ii }}$ | 2.182 (3) | $\mathrm{P}-\mathrm{O} 2^{\text {iii }}$ | 1.464 (4) |
| $\mathrm{Lu}-\mathrm{O} 3$ | 2.182 (3) | $\mathrm{P}-\mathrm{O} 3$ | 1.481 (4) |
| $\mathrm{Lu}-\mathrm{O} 2^{\text {i }}$ | 2.185 (4) | $\mathrm{P}-\mathrm{Ol}^{\text {iv }}$ | 1.583 (3) |
| $\mathrm{Lu}-\mathrm{O} 2{ }^{\text {ii }}$ | 2.185 (4) | $\mathrm{P}-\mathrm{O} 1$ | 1.594 (3) |
| Symmetry $-z+1,-$ | $\begin{aligned} & +1, x-\frac{1}{2} \\ & x-\frac{3}{4},-z \end{aligned}$ | $-\frac{1}{2} ; \quad \text { (ii) }$ | -1; (iii) |

Data collection: APEX2 (Bruker, 2008); cell refinement: SAINT (Bruker, 2008); data reduction: SAINT; program(s) used to solve structure: SHELXS97 (Sheldrick, 2008); program(s) used to refine structure: SHELXL97 (Sheldrick, 2008); molecular graphics: CaRine (Boudias \& Monceau, 1998) and ORTEP-3 (Farrugia, 1997); software used to prepare material for publication: SHELXTL (Sheldrick, 2008).

Supplementary data and figures for this paper are available from the IUCr electronic archives (Reference: WM2342).

## References

Bagieu-Beucher, M. (1976). J. Appl. Cryst. 9, 368-369.
Bagieu-Beucher, M. \& Guitel, J. C. (1978). Acta Cryst. B34, 1439-1442.
Bejaoui, A., Horchani-Naifer, K. \& Férid, M. (2008). Acta Cryst. E64, 148.
Boudias, C. \& Monceau, D. (1998). CaRine. CaRine Crystallography, DIVERGENT S.A., Compiègne, France.
Bruker (2008). APEX2, SAINT and SADABS. Bruker AXS Inc., Madison, Wisconsin, USA.
Chudinova, N. N. (1979). Izv. Akad. Nauk SSSR Neorg. Mater. 15, 833-837.
Durif, A. (1995). In Crystal Chemistry of Condensed Phosphates. New York and London: Plenum Press.
d'Yvoire, F. (1962). Bull. Soc. Chim. pp. 1237-1243.
Farrugia, L. J. (1997). J. Appl. Cryst. 30, 565.
Flack, H. D. (1983). Acta Cryst. A39, 876-881.
Hendricks, S. B. \& Wyckoff, R. W. G. (1927). Am. J. Sci. 13, 491-496.
Höppe, H. A. \& Sedlmaier, S. J. (2007). Inorg. Chem. 46, 3467-3474.
Liebau, F. \& Williams, H. P. (1964). Angew. Chem. 76, 303-304.
Mezentseva, L. P., Domanskii, A. I. \& Bondar, I. A. (1977). Russ. J. Inorg. Chem. 22, 43-45.
Pauling, L. \& Sherman, J. S. (1937). Z. Kristallogr. 96, 481-487.
Rémy, P. \& Boullé, A. (1964). C. R. Acad. Sci. 258, 927-929.
Sheldrick, G. M. (2008). Acta Cryst. A64, 112-122.
Smolin, Y. I., Shepelev, Y. F., Domanskii, A. I. \& Belov, N. V. (1978). Kristallografiya, 23, 187-188.
Yuan, J. L., Zhang, H., Zhao, J. T., Chen, H. H., Yang, X. X. \& Zhang, G. B. (2008). Opt. Mater. 30, 1369-1374.

## supplementary materials

## Lutetium(III) cyclotetraphosphate

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## Comment

The title compound is the third polymorph of composition $\mathrm{Lu}\left(\mathrm{PO}_{3}\right)_{3}$ besides the monoclinic form described by Höppe \& Sedlmaier (2007) and Yuan et al. (2008) and the trigonal form more recently reported by Bejaoui et al. (2008). The title compound is also the less dense polymorph with a calculated density of $3.451 \mathrm{Mg}^{\prime} \mathrm{m}^{3}$ versus $3.587 \mathrm{Mg}^{\prime} \cdot \mathrm{m}^{3}$ for the trigonal and $3.708 \mathrm{Mg}^{3}$ for the monoclinic form and is probably the highest temperature form. This cyclotetraphosphate belongs to a structural type (cubic, with space group $\overline{\overline{4}} 3 \mathrm{~d}$ ) known since 1927 through the archetype $\mathrm{Al}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ determined by Hendricks \& Wyckoff (1927). Then Pauling \& Sherman (1937) gave the first description of the structure and reported roughly estimated atomic coordinates deduced from geometrical considerations. Since this time only five members of this family, viz. $\mathrm{Cr}_{4}{ }^{\mathrm{III}}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ (Rémy \& Boullé, 1964), $\mathrm{Ti}_{4}{ }^{\mathrm{III}}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ (Liebau \& Williams, 1964), $\mathrm{Fe}_{4}{ }^{\mathrm{III}}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ (d'Yvoire et al., 1962), $\mathrm{Sc}_{4}{ }^{\mathrm{III}}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ (Bagieu-Beucher, 1976; Mezentseva et al., 1977 and Smolin et al., 1978) and $\mathrm{Yb}_{4}{ }^{\mathrm{III}}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ (Chudinova, 1979), have been identified. Corresponding unit cell parameters are listed in Durif (1995). Among these isotypic compounds only the structure of the $\mathrm{Sc}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ cyclotetraphosphate has almost simultaneously been refined from single-crystal data by Bagieu-Beucher \& Guittel (1978) and Smolin et al. (1978). Their refinements confirmed the description of Pauling \& Sherman (1937) according to which all the crystallographically independent atoms except the $A^{\text {III }}$ element (.3. symmetry) are in general positions. The structure is built of four-membered phosphate ring anions $\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)^{4-}$ (Fig. 1), isolated from each other and further linked by $\mathrm{LuO}_{6}$ octahedra by sharing corners. Each $\mathrm{LuO}_{6}$ octahedron is linked to six $\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)^{4-}$ rings (Fig. 2a) while each $\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)^{4-}$ ring is linked to eight $\mathrm{LuO}_{6}$ octahedra (Fig. 2b) through oxygen atoms with shorter $\mathrm{P}-\mathrm{O}$ distances $\left(1.464\right.$ (4) and 1.481 (4) $\AA$ ). The $\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)^{4-}$ ring anions are located around the 12 a Wyckoff positions of space group $\overline{4} 3 d$ and exhibit $\overline{4}$ symmetry. Comparison of the $\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)^{4-}$ ring anions in both $\mathrm{Sc}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ and $\mathrm{Lu}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ structures shows these two ring anions being geometrically quite identical with alternating upward- and downward-pointing tetrahedra and $\mathrm{P}-\mathrm{O}-\mathrm{P}$ angles of $137.1^{\circ}$ and $136.9(2)^{\circ}$, respectively. The $\mathrm{P}-\mathrm{O}$ distances in the $\mathrm{PO}_{4}$ groups are identical within their e.s.d.. The four bridging oxygen atoms of these ring anions are located at the apices of a flattened tetrahedron with characteristic angles of $148.22^{\circ}$ and $94.30^{\circ}$ for Sc and $147.95^{\circ}$ and $94.37^{\circ}$ for the Lu cyclotetraphosphate. $\mathrm{The}^{\mathrm{LuO}} 6$ octahedron is very slightly distorted along a threefold axis, resulting in two sets of $\mathrm{Lu}-\mathrm{O}$ distances equal to 2.182 (3) and 2.185 (4) Å, respectively.

## Experimental

Single crystals of the title compound were obtained by solid state reaction while attempting to synthesized a long chain polyphosphate by reacting $\mathrm{Lu}_{2} \mathrm{O}_{3}$ with $\left(\mathrm{NH}_{4}\right) \mathrm{H}_{2} \mathrm{PO}_{4}$ and $\mathrm{Rb}_{2} \mathrm{CO}_{3}$ in an alumina boat. A mixture of these reagents in the molar ratio $27: 85.5: 8.7$ was used for the synthesis. The mixture was successively heated at 473 K for 24 hours, then at 573 K for 24 additional hours and finally at 813 K for 24 hours. Then the sample was cooled down to 683 K at the rate of $3 \mathrm{~K} \mathrm{~h}^{-1}$ and maintained at this temperature for 36 hours. Finally, the sample was cooled down to room temperature by shutting the

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muffle furnace off. Single crystals were extracted from the batch by washing with hot water and filtering. The crystals were dried at 353 K in an oven. A translucent octahedral crystal of the title compound was selected for the structure refinement.

## Refinement

The highest residual peak in the final difference Fourier map was located $0.87 \AA$ from atom Lu and the deepest hole was located $0.99 \AA$ from atom Lu.

## Figures



Fig. 1. ORTEP-3 view of the four-membered phosphate $\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)^{4-}$ ring anion. Displacement ellipsoids are drawn at the $50 \%$ probability level. Symmetry codes: (i) $1-z, 3 / 2-x, y$; (ii) $3 /$ $4+y, 5 / 4-x, 3 / 4-z$; (iii) $5 / 4-y,-3 / 4+x, 3 / 4-z$; (iv) $-1 / 4+x, 1 / 4-z, 3 / 4-y$; (v) $9 / 4-x, 1 / 4+z, 3 / 4-y$ ;(vi) $2-x, 1 / 2-y, z$; (vii) $1+z,-1+x, y$.


Fig. 2. Partial view of the $\mathrm{Lu}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$ structure showing: (a) the connections between the $\mathrm{LuO}_{6}$ octahedron and the $\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)^{4-}$ ring anions, (b) the connections between the $\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)^{4-}$ ring anion and the $\mathrm{LuO}_{6}$ octahedra.

## Tetralutetium(III) tris(cyclotetraphosphate)

## Crystal data

$\mathrm{Lu}_{4}\left(\mathrm{P}_{4} \mathrm{O}_{12}\right)_{3}$
$M_{r}=1647.52$

Cubic, $\sqrt{4} 3$ d
Hall symbol: I -4bd 2c 3
$a=14.6920$ (6) $\AA$
$V=3171.3(2) \AA^{3}$
$Z=4$
$F(000)=3008$

## Data collection

## Bruker APEXII CCD

diffractometer
Radiation source: fine-focus sealed tube graphite
Detector resolution: 8.3333 pixels $\mathrm{mm}^{-1}$
$\varphi$ and $\omega$ scans
Absorption correction: multi-scan
(SADABS; Bruker, 2008)
$T_{\text {min }}=0.534, T_{\max }=0.746$
$D_{\mathrm{x}}=3.451 \mathrm{Mg} \mathrm{m}^{-3}$
Mo $K \alpha$ radiation, $\lambda=0.71073 \AA$
Cell parameters from 1548 reflections
$\theta=3.4-30.3^{\circ}$
$\mu=13.08 \mathrm{~mm}^{-1}$
$T=296 \mathrm{~K}$
Truncated octahedron, colourless
$0.18 \times 0.10 \times 0.08 \mathrm{~mm}$

$$
\begin{aligned}
& 717 \text { independent reflections } \\
& 659 \text { reflections with } I>2 \sigma(I) \\
& R_{\mathrm{int}}=0.034 \\
& \theta_{\max }=30.4^{\circ}, \theta_{\min }=3.9^{\circ} \\
& h=-16 \rightarrow 11 \\
& k=-6 \rightarrow 20 \\
& l=-19 \rightarrow 9
\end{aligned}
$$

3088 measured reflections

## Refinement

## Refinement on $F^{2}$

Least-squares matrix: full
$R\left[F^{2}>2 \sigma\left(F^{2}\right)\right]=0.020$
$w R\left(F^{2}\right)=0.038$
$S=1.03$
717 reflections
41 parameters
0 restraints
0 constraints

Primary atom site location: structure-invariant direct methods
Secondary atom site location: difference Fourier map
$w=1 /\left[\sigma^{2}\left(F_{\mathrm{o}}{ }^{2}\right)+(0.0088 P)^{2}\right]$
where $P=\left(F_{\mathrm{o}}{ }^{2}+2 F_{\mathrm{c}}{ }^{2}\right) / 3$
$(\Delta / \sigma)_{\text {max }}<0.001$
$\Delta \rho_{\max }=0.90$ e $\AA^{-3}$
$\Delta \rho_{\text {min }}=-0.67$ e $\AA^{-3}$
Absolute structure: Flack (1983), 272 Friedel pairs
Flack parameter: 0.000 (15)

## Special details

Geometry. All esds (except the esd in the dihedral angle between two 1.s. planes) are estimated using the full covariance matrix. The cell esds are taken into account individually in the estimation of esds in distances, angles and torsion angles; correlations between esds in cell parameters are only used when they are defined by crystal symmetry. An approximate (isotropic) treatment of cell esds is used for estimating esds involving 1.s. planes.
Refinement. Refinement of $F^{2}$ against ALL reflections. The weighted $R$-factor wR and goodness of fit $S$ are based on $F^{2}$, conventional $R$-factors $R$ are based on $F$, with $F$ set to zero for negative $F^{2}$. The threshold expression of $F^{2}>\sigma\left(F^{2}\right)$ is used only for calculating $R$ factors(gt) etc. and is not relevant to the choice of reflections for refinement. $R$-factors based on $F^{2}$ are statistically about twice as large as those based on $F$, and $R$ - factors based on ALL data will be even larger.

Fractional atomic coordinates and isotropic or equivalent isotropic displacement parameters $\left(A^{2}\right)$

|  | $x$ | $y$ | $z$ | $U_{\text {iso }} * / U_{\text {eq }}$ |
| :--- | :--- | :--- | :--- | :--- |
| Lu | $0.896610(13)$ | $0.396610(13)$ | $0.103390(13)$ | $0.00602(8)$ |
| P | $0.95737(9)$ | $0.37294(9)$ | $0.33447(9)$ | $0.0077(2)$ |
| O1 | $1.0613(2)$ | $0.3432(2)$ | $0.3430(2)$ | $0.0128(7)$ |
| O2 | $1.0325(2)$ | $0.3642(3)$ | $0.0522(2)$ | $0.0188(8)$ |
| O3 | $0.9295(2)$ | $0.3498(3)$ | $0.2404(2)$ | $0.0142(7)$ |

Atomic displacement parameters $\left(A^{2}\right)$

|  | $U^{11}$ | $U^{22}$ | $U^{33}$ | $U^{12}$ | $U^{13}$ | $U^{23}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Lu | $0.00602(8)$ | $0.00602(8)$ | $0.00602(8)$ | $0.00039(8)$ | $-0.00039(8)$ | $-0.00039(8)$ |
| P | $0.0097(5)$ | $0.0051(6)$ | $0.0082(6)$ | $0.0017(5)$ | $-0.0008(5)$ | $0.0015(4)$ |
| O 1 | $0.0113(15)$ | $0.0146(18)$ | $0.0123(17)$ | $0.0007(15)$ | $-0.0025(14)$ | $0.0054(17)$ |
| O 2 | $0.0103(17)$ | $0.024(2)$ | $0.022(2)$ | $0.0016(18)$ | $0.0015(16)$ | $-0.0014(18)$ |
| O 3 | $0.0202(19)$ | $0.0141(19)$ | $0.0084(16)$ | $0.0012(16)$ | $-0.0050(15)$ | $0.0023(16)$ |

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Geometric parameters ( $A,{ }^{\circ}$ )

| $\mathrm{Lu}-\mathrm{O} 3^{\text {i }}$ | 2.182 (3) | $\mathrm{P}-\mathrm{O} 2^{\text {iii }}$ | 1.464 (4) |
| :---: | :---: | :---: | :---: |
| $\mathrm{Lu}-\mathrm{O} 3^{\text {ii }}$ | 2.182 (3) | $\mathrm{P}-\mathrm{O} 3$ | 1.481 (4) |
| $\mathrm{Lu}-\mathrm{O} 3$ | 2.182 (3) | $\mathrm{P}-\mathrm{Ol}^{\text {iv }}$ | 1.583 (3) |
| $\mathrm{Lu}-\mathrm{O} 2{ }^{\text {i }}$ | 2.185 (4) | $\mathrm{P}-\mathrm{O} 1$ | 1.594 (3) |
| $\mathrm{Lu}-\mathrm{O} 2{ }^{\text {ii }}$ | 2.185 (4) | O1-P ${ }^{\text {v }}$ | 1.583 (3) |
| $\mathrm{Lu}-\mathrm{O} 2$ | 2.185 (4) | $\mathrm{O} 2-\mathrm{P}^{\mathrm{vi}}$ | 1.464 (4) |
| $\mathrm{O} 3{ }^{\mathrm{i}}-\mathrm{Lu}-\mathrm{O} 3{ }^{\text {ii }}$ | 89.06 (14) | $\mathrm{O} 3-\mathrm{Lu}-\mathrm{O} 2$ | 92.64 (13) |
| $\mathrm{O} 3{ }^{\mathrm{i}}-\mathrm{Lu}-\mathrm{O} 3$ | 89.06 (14) | $\mathrm{O} 2{ }^{\mathrm{i}}-\mathrm{Lu}-\mathrm{O} 2$ | 87.72 (15) |
| $\mathrm{O} 3{ }^{\text {ii }}-\mathrm{Lu}-\mathrm{O} 3$ | 89.06 (14) | $\mathrm{O} 2{ }^{\text {ii }}-\mathrm{Lu}-\mathrm{O} 2$ | 87.72 (15) |
| $\mathrm{O} 3{ }^{\mathrm{i}}-\mathrm{Lu}-\mathrm{O} 2^{\text {i }}$ | 92.64 (13) | $\mathrm{O} 2{ }^{\text {iiii }}$-P-O3 | 118.0 (2) |
| $\mathrm{O} 3{ }^{\text {ii }}-\mathrm{Lu}-\mathrm{O} 2^{\text {i }}$ | 178.26 (14) | $\mathrm{O} 2{ }^{\text {iii }}-\mathrm{P}-\mathrm{O} 1^{\text {iv }}$ | 107.3 (2) |
| $\mathrm{O} 3-\mathrm{Lu}-\mathrm{O}^{\text {i }}$ | 90.59 (14) | $\mathrm{O} 3-\mathrm{P}-\mathrm{O}^{\text {iv }}$ | 111.5 (2) |
| $\mathrm{O} 3{ }^{\text {i }}-\mathrm{Lu}-\mathrm{O} 2^{\mathrm{ii}}$ | 90.59 (14) | $\mathrm{O} 2{ }^{\text {iii }}-\mathrm{P}-\mathrm{O} 1$ | 109.2 (2) |
| $\mathrm{O} 3{ }^{\text {ii }}-\mathrm{Lu}-\mathrm{O} 22^{\text {ii }}$ | 92.64 (13) | $\mathrm{O} 3-\mathrm{P}-\mathrm{O} 1$ | 106.0 (2) |
| $\mathrm{O} 3-\mathrm{Lu}-\mathrm{O} 2{ }^{\text {ii }}$ | 178.26 (14) | $\mathrm{O} 1{ }^{\text {iv }}-\mathrm{P}-\mathrm{O} 1$ | 103.9 (2) |
| $\mathrm{O} 2 \mathrm{i}-\mathrm{Lu}-\mathrm{O} 2^{\mathrm{ii}}$ | 87.72 (15) | $\mathrm{P}^{\mathrm{v}}$-O1-P | 136.9 (2) |
| $\mathrm{O} 3{ }^{\mathrm{i}}-\mathrm{Lu}-\mathrm{O} 2$ | 178.26 (14) | $\mathrm{P}^{\mathrm{vi}}-\mathrm{O} 2-\mathrm{Lu}$ | 164.9 (2) |
| $\mathrm{O} 3{ }^{\text {ii }}-\mathrm{Lu}-\mathrm{O} 2$ | 90.59 (14) | $\mathrm{P}-\mathrm{O} 3-\mathrm{Lu}$ | 148.2 (2) |

Symmetry codes: (i) $-z+1, x-1 / 2,-y+1 / 2$; (ii) $y+1 / 2,-z+1 / 2,-x+1$; (iii) $-z+1,-x+3 / 2, y$; (iv) $-y+5 / 4, x-3 / 4,-z+3 / 4$; (v) $y+3 / 4,-x+5 /$ $4,-z+3 / 4$; (vi) $-y+3 / 2, z,-x+1$.

Fig. 1


Fig. 2


